The Arithmetic Problem

Comments on K-8 Mathematics Curriculum and Assessment

Steve Newman

I. Overview

Too many of our students do not know their multiplication tables, far too many are cannot add, subtract, multiply and divide fractions and decimals, too many lack number sense and cannot distinguish reasonable answers from unreasonable ones, and too many are totally dependent on calculators for even the simplest calculations. This in a nutshell is The Arithmetic Problem that threatens the future of our state and our nation.

The National Mathematics Advisory Panel and the National Council of Teachers of Mathematics are two prominent national groups that have sounded the alarm about this problem and have provided remarkably similar recommendations for change that are consistent with what is being done in industrialized nations that out perform the United States in mathematics and science.

The purpose of this paper is to highlight the findings and recommendations of these two groups and to promote discussion of these findings and recommendations among policy makers, education leaders, school administrators, teachers, parents and interested citizens. The paper is specifically directed to members of the Task Force on Assessment and Accountability. I have listened with great interest to the webcast of the first two meetings. I fully understand that the intent of the task force is to discuss assessment, not curriculum.

But the case for change in elementary and middle school mathematics curricula is so compelling that task force members should be aware of the recommendations of these national groups so that fully informed decisions can be made.

Policy makers and education leaders must ultimately decide whether the case for change is sufficiently compelling to begin implementation of the recommended changes now or whether Kentucky should stay the course until 2014 and risk falling even further behind other states and nations in mathematics achievement.

II. The National Mathematics Advisory Panel

The final report of the National Mathematics Advisory Panel was issued in March http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf. A compelling case for change is spelled out in the Executive Summary:

During most of the 20th century, the United States possessed peerless mathematical prowess-not just measured by the number of mathematical specialists who practiced here but also by the scale and quality of its engineering, science, and financial leadership, and even the extent of mathematical education in its broad population. But without substantial and sustained changes to its educational system, the United States will relinquish its leadership in the 21st century. This report is about actions that must be taken to strengthen the American people in this central area of learning. Success matters

to the nation at large. It matters, too, to individual students and their families, because it opens doors and creates opportunities.

This Panel, diverse in experience, expertise, and philosophy, agrees broadly that the delivery system in mathematics education-the system that translates mathematical knowledge into value and ability for the next generation- is broken and must be fixed. This is not a conclusion about any single element in the system. It is about how the many parts do not now work together to achieve a result of this country's values and ambitions.

The Panel expressed particular concern about the nation's ability to produce a sufficient number of students in the science, technology, engineering, and mathematics (STEM) disciplines and its reliance on importing this technical talent. The Panel viewed this as a threat to our economy and to our national security.

There are consequences to a weakening of U.S. independence and leadership in mathematics, the natural sciences, and engineering. Looking at the fast pace of technological advancement in the United States, Schacht (2005) commented, "It is widely accepted that technological progress is responsible for up to one-half the growth of the U.S. economy, and is one principal driving force in long-term growth and living standards." Ignoring threats to the nation's ability to advance in the science, technology, engineering, and mathematics (STEM) fields will put our economic viability and our basis for security at risk.

Algebra is seen as a central concern because it is a demonstrable gateway to later achievement. Thus the Panel recommended focusing the elementary and middle school mathematics curriculum on what it calls the Critical Foundations of Algebra. These foundations consist of three clusters of concepts and skills: fluency with whole numbers, fluency with fractions, decimals, and percents, and fluency with specific aspects of geometry and measurement. The Panel's Benchmarks for Critical Foundations of Algebra in *Appendix A* spell out the foundational topics and when they should be taught.

The Panel found that fractions are of particular concern:

Difficulty with fractions (including decimals and percents) is pervasive and is a major obstacle to further progress in mathematics, including algebra. A nationally representative sample of teachers of Algebra I who were surveyed for the Panel rated students as having very poor preparation in "rational numbers and operations involving fractions and decimals."

The Panel felt that mathematics education in this country should be made more coherent and recommended that fewer topics be taught in greater depth at each grade level as is done in top-performing industrialized nations that consistently out perform the United States on international tests. The Panel felt that topics in the Critical Foundations of Algebra should be given greater emphasis and should be taught in greater depth.

The Panel noted two major differences between the curricula in top-performing countries and the United States:

There seem to be two major differences between the curricula in top-performing countries and those in the U.S.-in the number of mathematical topics presented at each grade level, with each receiving only limited development, while top-performing countries present fewer topics at each grade level but in greater depth. In addition, U.S. curricula generally review and extend at successive grade levels many (if not most) topics already presented at earlier grade levels, while the top performing countries are more likely to expect closure after exposure, development, and refinement of a particular topic.

II. The National Council of Teachers of Mathematics

The National Council of Teachers of Mathematics (NCTM) publication, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (http://my.nctm.org/ebusiness/ProductCatalog/product.aspx?ID=13089) was issued in 2006. The problems faced by mathematics teachers is succinctly summarized in the preface:

As state and local school districts implement more rigorous assessment and accountability systems, teachers often face long lists of mathematics topics or learning expectations to address at each grade level, with many topics repeating from year to year. Lacking clear, consistent priorities and focus, teachers stretch to find the time to present important mathematical topics effectively and in depth.

The NCTM calls for coherence in the curriculum by focusing in depth on a small number of essential mathematical topics at each grade level and for a move away from the "mile-wide, inch-deep" curriculum so prevalent in the United States.

The focal points consist of only three major learning objectives at each grade level from pre-kindergarten through grade 8. The focal points for grades 1 through 8 are in *Appendix B.*

The alignment between what the NCTM recommends in the focal points and what the National Mathematics Advisory Panel recommends in the Benchmarks for Critical Foundations of Algebra is stunning.

A clear and powerful national consensus has been reached on the essential mathematical topics that should be taught in grades 1 through 8 and when these topics should be taught. This consensus is closely aligned with what is already being done internationally in the top-performing countries.

III. The Situation in Kentucky

The Arithmetic Problem is at least as serious in Kentucky as it is in other states.

Algebra I teachers in Kentucky would undoubtedly rate their students as having *very poor preparation* in "rational numbers and operations involving fractions and decimals" just as their peers nationwide did in the survey conducted by the National Mathematics Advisory Panel. These teachers know all too well how difficult it is to try to teach algebra to students who do not know arithmetic.

Difficulties with arithmetic lead to difficulties with algebra, and these difficulties often persist throughout high school and into postsecondary education and the workforce.

No one doubts that Kentucky has a serious problem in mathematics. Kentucky students consistently score much further below the national average on the ACT in mathematics than they do in English, reading and science. The data in Table 1 illustrate the serious problem we face in Kentucky. It shows the percentage of Kentucky students at or above the ACT benchmark score on each of the four ACT sub-tests in 2007. The benchmark score in mathematics is 22. What this means is that students who score at or above 22 on the math ACT test have a good chance of earning a grade of "C" or better in college algebra, a conclusion based on extensive ACT research. The benchmark scores in science, reading and English are determined in a similar manner.

The data show that Kentucky is significantly behind the nation in mathematics, with only 35% of our students prepared for college algebra as opposed to 43% nationwide. Since a strong background in mathematics is critically important for students who intend to major in a STEM discipline, this glaring deficiency is not a good sign. Kentucky is lagging well behind a nation that is lagging well behind the industrialized world!

Table 1
2007 ACT Data

ACT Sub-Test	ACT Benchmark Score	National % at or above Benchmark	Kentucky % at or above Benchmark	Kentucky % behind the Nation
Mathematics	22	43	35	8
Science	24	28	24	4
Reading	21	53	50	3
English	18	69	67	2

It is ironic that despite poor preparation of Kentucky students in arithmetic, the CATS scores in mathematics are significantly *higher* in elementary school than in middle school or high school. Table 2 gives the CATS scores in mathematics at the elementary, middle, and high school levels as projected for 2014. These projections were discussed at the June 2007 meeting of the Kentucky Board of Education.

Table 2
CATS Scores Projected for 2014 in Mathematics

	Elementary School	Middle School	High School
Mathematics	105.4	85.0	84.0

Elementary schools statewide are projected to be well above the proficiency level of 100, whereas middle schools and high schools are projected to be well below. In particular, there is a steep decline in scores of over 20 points from the elementary to the middle school level.

One of the possible interpretations of this steep decline is a lack of alignment in what students learn in mathematics in the elementary grades compared with what is expected in the middle and high school grades. It appears that many students are proficient in mathematics in grade 5 as measured by the CATS test and yet are not prepared for middle and high school mathematics.

IV. Kentucky Curriculum and Assessments

Kentucky teachers often express concern about covering too many topics in too little time in order to prepare their students for the CATS assessment and thus not being able to cover essential topics effectively and in depth. They understand, as do their peers nationwide, that state assessment and accountability systems unintentionally tend to produce and promote the "mile wide, inch deep" curriculum so common in this country. This is the problem that the National Council of Teachers of Mathematics and the National Mathematics Advisory Panel hoped to solve in K-8 mathematics by developing the focal points and the critical foundations of algebra.

A comparison of Kentucky's Core Content for Assessment in Grade 5 Mathematics (*Appendix C*) with recommendations made by these national groups makes clear the problem facing Kentucky's elementary school teachers in their efforts to solve The Arithmetic Problem. The core content is composed of five strands: Number Properties and Operations, Measurement, Geometry, Data Analysis and Probability, and Algebraic Thinking. The topics in Number Properties and Operations, together with certain topics in Geometry and Measurement, are *by far* the most important and are the only topics mentioned as focal points or as critical foundations of algebra in grades 1 through 5. There are *no topics* in Data Analysis and Probability or in Algebraic Thinking mentioned as focal points or as critical foundations of algebra in grades 1 through 5, although a few are mentioned in grades 6 through 8.

The core content in mathematics for grade 5 is particularly weak in its standards relating to fractions, the very topic most emphasized by the national groups. Among other weaknesses, MA-05-1.3.1 in *Appendix C* states

- add and subtract fractions with like denominators through 16, with sums less than or equal to one

Adding and subtracting fractions with *any* denominators and *any* sums, and the concept of a common denominator, is recommended in grade 5 as a focal point and as a critical foundation of algebra.

The lack of focus on the essential topics in the core content in mathematics for grade 5 is especially unfortunate because elementary school teachers are generalists who teach all subjects. They are less able to distinguish important topics in mathematics from less important ones. Even if they were made aware of these distinctions through teacher preparation courses and professional development, they would still need to teach the wide range of topics in the core content to prepare their students for the CATS test and would still struggle to teach the essential topics effectively and in depth.

The point is that the curriculum cannot be focused on essential topics unless the CATS assessment test is also focused on these topics. Curriculum and assessment are inextricably linked.

There is, however, an assessment initiative that has promise in focusing more attention on The Arithmetic Problem. *A portion of the CATS test in elementary and middle school mathematics could be done without a calculator*. This change would send a strong message that Kentucky is serious about improving the arithmetic skills of its students. It is all too easy for Kentucky teachers, pressed for time as they are, to give minimal coverage to essential arithmetic skills because they realize that students will be able to use their calculators on the CATS test. The depressing results of this minimal coverage are clear to middle school, high school, and postsecondary mathematics and science teachers as well as to employers.

V. Conclusion.

Kentucky should follow the recommendations of the National Mathematics Advisory Panel and the National Council of Teachers of Mathematics. These recommendations call for fewer topics to be covered in elementary and middle school mathematics so that a smaller number of clearly specified, essential topics can be covered in greater depth. This is the best way, and likely the only way, we can solve The Arithmetic Problem.

These recommendations reflect what is done in top performing industrialized countries that consistently out perform the United States on international tests in mathematics and science. We cannot compete, either as a state or as a nation, with these countries in the knowledge-based, global economy of the 21st century unless we improve our system of mathematics education.

We as a state cannot afford to ignore these recommendations that represent such a powerful national and international consensus. We must not delay their implementation. We cannot wait until 2014; we must act now. Our future depends upon it.

Appendix A
National Mathematics Advisory Panel Report

Benchmarks for Critical Foundations of Algebra

Fluency With Whole Numbers

1) By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.
2) By the end of Grade 5, students should be proficient with the multiplication and division of whole numbers.

Fluency With Fractions

1) By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.
2) By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percent, and with the addition and subtraction of fractions and decimals.
3) By the end of Grade 6, students should be proficient with the multiplication and division of fractions and decimals.
4) By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.
5) By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.
6) By the end of grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.

Geometry and Measurement

1) By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
2) By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.
3) By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of a slope of a line.

Appendix B

National Council of Teachers of Mathematics (NCTM) Focal Points

Grade 1

- **Number Operations and Algebra:** Developing understandings of addition and subtraction strategies for basic addition facts and related subtraction facts

- **Number and Operations:** Developing an understanding of whole number relationships, including grouping in tens and ones
- **Geometry:** Composing and Decomposing geometric shapes

Grade 2

- **Number and Operations:** Developing an understanding of the base-ten numeration system and place value concepts
- **Number Operations and Algebra:** Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction
- **Measurement:** Developing an understanding of linear measurement and facility in measuring lengths

Grade 3

- **Number Operations and Algebra:** Developing understandings of multiplication and division and strategies for basic multiplication facts and related division facts
- **Number and Operations:** Developing an understanding of fractions and fraction equivalence
- **Geometry:** Developing and analyzing properties of two dimensional shapes

Grade 4

- **Number Operations and Algebra:** Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication
- **Number and Operations:** Developing an understanding of decimals, including the connections between fractions and decimals
- **Measurement:** Developing an understanding of area and determining the areas of two-dimensional figures

Grade 5

- **Number Operations and Algebra:** Developing an understanding of and fluency with division of whole numbers
- **Number and Operations:** Developing an understanding of and fluency with addition and subtraction of fractions and decimals
- **Geometry and Measurement and Algebra:** Describing three-dimensional shapes and analyzing their properties, including volume and surface area

Grade 6

- **Number and Operations:** Developing and understanding of and fluency with multiplication and division of fractions and decimals
- **Number and Operations:** Connecting ratio and rate to multiplication and division
- **Algebra:** Writing, interpreting, and using mathematical expressions and equations

Grade 7

- **Number and Operations and Algebra and Geometry:** Developing and understanding of and applying proportionality, including similarity
- **Measurement and Geometry and Algebra:** Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes
- **Number and Operations and Algebra:** Developing an understanding of operations on all rational numbers and solving linear equations

Grade 8

- **Algebra:** Analyzing and representing linear functions and solving linear equations and systems of linear equations
- **Geometry and Measurement:** Analyzing two- and three-dimensional space and figures by using distance and angle
- **Data Analysis and Number Operations and Algebra:** Analyzing and summarizing data sets

Appendix C

Grade 5 Core Content for (Statewide) Assessment in Mathematics

Number Properties and Operations

MA-05-1.1.1
Students will:

- apply multiple representations (e.g., drawings, manipulatives, base-10 blocks, number lines, expanded form, symbols) to represent whole numbers (0 to 99,999,999);
- apply multiple representations (e.g., drawings, manipulatives, base-10 blocks, number lines, symbols) to describe commonly-used fractions, mixed numbers and decimals through thousandths;
- apply these numbers to represent real-world problems and explain how the base-10 number system relates to place value.

MA-05-1.1.3

Students will compare (<, >, =) and order whole numbers), fractions and decimals, and explain the relationships (equivalence, order) between and among them.

MA-05-1.2.1

Students will apply and describe appropriate strategies for estimating quantities of objects and computational results in real-world problems.

MA-05-1.3.1
Students will analyze real-world problems to identify appropriate representations using mathematical operations, and will apply operations to solve real-world problems with the following constraints:

- add, subtract, multiply, and divide whole numbers (less than 100,000,000), using technology where appropriate;
- add and subtract fractions with like denominators through 16, with sums less than or equal to one and
- add and subtract decimals through hundredths.

MA-05-1.5.1
Students will identify and determine composite numbers, prime numbers, multiples of a number, factors of a number and least common multiples (LCM), and will apply these numbers to solve real-world problems.

Measurement

MA-05-2.1.1
Students will apply standard units to measure length (to the nearest eighth-inch or the nearest centimeter) and to determine:

- weight (ounce, pound; gram, kilogram);
- perimeter;
- area (figures that can be divided into rectangular shapes);
- time (nearest minute);
- temperature (Fahrenheit and Celsius) and
- angle measures (nearest degree).

MA-05-2.1.6
Students will estimate weight, length, perimeter, area, angle measures and time using appropriate units of measurement.

MA-05-2.2.1
Students will determine elapsed time.

MA-05-2.2.3
Students will convert units within the same measurement system [U.S. customary (inches, feet, yards, miles; ounces, pounds, tons), metric (millimeters, centimeters, meters, kilometers; grams, kilograms), money, or time] and use the units to solve problems.

Geometry

MA-05-3.1.1

Students will describe and provide examples of basic geometric elements and terms [points, segments, lines (perpendicular, parallel, intersecting), rays, angles (acute, right, obtuse), sides, edges, faces, bases, vertices, radius, diameter] and will apply these elements to solve real-world and mathematical problems.

MA-05-3.1.2
Students will describe and provide examples of basic two-dimensional shapes [circles, triangles (right, equilateral), all quadrilaterals, pentagons, hexagons, octagons] and will apply these shapes to solve real-world and mathematical problems.

MA-05-3.1.3
Students will describe and provide examples of basic three-dimensional objects (spheres, cones, cylinders, pyramids, cubes, triangular and rectangular prisms), will identify three-dimensional objects from two-dimensional representations (nets) and will apply the attributes to solve real-world and mathematical problems.

MA-05-3.1.5
Students will identify and describe congruent and similar figures in real-world and mathematical problems.

MA-05-3.2.1
Students will describe and provide examples of line symmetry in real-world and mathematical problems or will apply line symmetry to construct a geometric design.

MA-05-3.2.2
Students will identify 90º rotations, reflections or translations of basic shapes within a plane.

MA-05-3.3.1
Students will identify and graph ordered pairs on a positive coordinate system scaled by ones, twos, threes, fives or tens; locate points on a grid; and apply graphing in the coordinate system to solve real-world problems.

Data Analysis and Probability

MA-05-4.1.1
Students will analyze and make inferences from data displays (drawings, tables/charts, tally tables, pictographs, bar graphs, circle graphs, line plots, Venn diagrams, line graphs).

MA-05-4.1.3
Students will construct data displays (pictographs, bar graphs, line plots, line graphs, Venn diagrams, tables).

MA-05-4.2.1
Students will determine and apply the mean, median, mode and range of a set of data.

MA-05-4.4.1
Students will determine all possible outcomes of an activity/event with up to 12 possible outcomes.

MA-05-4.4.2
Students will determine the likelihood of an event and the probability of an event (expressed as a fraction).

Algebraic Thinking

MA-05-5.1.1
Students will extend patterns, find the missing term(s) in a pattern or describe rules for patterns (numbers, pictures, tables, words) from real-world and mathematical problems.

MA-05-5.1.2
Students will describe functions (input-output) through pictures, tables, or words and will construct tables to analyze functions based on real-world or mathematical problems.

MA-05-5.1.3
Students will determine an output value or an input value for a function rule given the other value.

MA-05-5.2.1
Students will model verbal descriptions of real-world and mathematical problems using a variable or a missing value in an expression.

MA-05-5.3.1
Students will model real-world and mathematical problems with simple number sentences (equations and inequalities) with a variable or missing value (e.g., 4 = 2 x N, ___+ 5 > 14) and apply simple number sentences to solve mathematical and real-world problems.

About the Author
Steve Newman is professor of mathematics at Northern Kentucky University, director of the Kentucky Early Mathematics Testing Program (KEMTP), and one of several leaders of the Kentucky Online Testing (KYOTE) placement-testing project. He has been involved in curriculum alignment issues in mathematics for over a decade. He served on the national Content Expert/Employer Panel that developed the American Diploma Project benchmarks in mathematics, the statewide group that developed the postsecondary placement policy and college readiness standards in mathematics, and the Commissioner's Task Force that developed the core content standards in high school mathematics. His recent service on the Mathematics Task Force in elementary school

mathematics for the Educational Professional Standards Board sharpened his interest in the elementary school mathematics curriculum.

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